Robust estimation of vehicle lateral velocity and yaw rate using Switched T-S Fuzzy Interval Observer

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Outline of the talk

- Context: estimation of vehicle lateral dynamics
- Problem Statement and Some Background
- System description
- Switched Interval Observer Design
- Experimental validation
- Conclusion

Context: estimation of vehicle lateral dynamics

- Knowledge of vehicle lateral velocity is essential for safety enhancement, in particular for development of active safety systems.

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- Direct measurement of lateral velocity requires the use of high cost sensors which cannot be used in production cars.
- As a consequence the virtual sensing approach (observer) proposed here may be of particular interest.
- *Goal*: Robust estimation process of vehicle lateral velocity and yaw rate taking into account: Model uncertainty and changes in operating conditions.

Problem statement

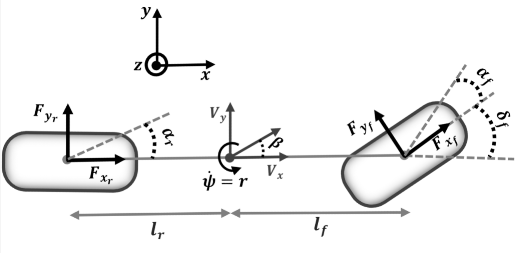

Figure: Bicycle Model.

- Reduces tires at each axle to a single equivalent tire;
- The vehicle state is described by body-fixed lateral velocity and yaw rate

Problem statement

The dynamics equations can be represented by (Rajamani(2011)):

$$\begin{cases} m\dot{v}_y + mr = F_{yf} + F_{yr} \\ I_z\dot{r} = l_f F_{yf} - l_r F_{yr} \end{cases} \tag{1}$$

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- m, I_z, l_r, l_f denote respectively the mass of the vehicle, the yaw moment and the distances from the rear and the front axle to the center of gravity.
- v_x is a time-varying longitudinal velocity, v_y is the lateral velocity of the vehicle and r is the yaw rate.
- F_{yr} and F_{yf} are the lateral rear and front forces respectively.

Problem statement

Using Pacejka's magic formula (Pacejka and Bakker (1991)), the lateral forces are given by:

$$F_{yi} = D_i sin(C_i tan^{-1}(B_i(1 - E_i)\alpha_i + E_i tan^{-1}(B_i\alpha_i))) \quad (2)$$

- $i = \{r, f\}$ denotes rear and front of the vehicle;
- D_i, C_i, B_i and E_i are the characteristic constants of the tires.
- α_f and α_r are respectively the front and rear sideslip angles of the tires.

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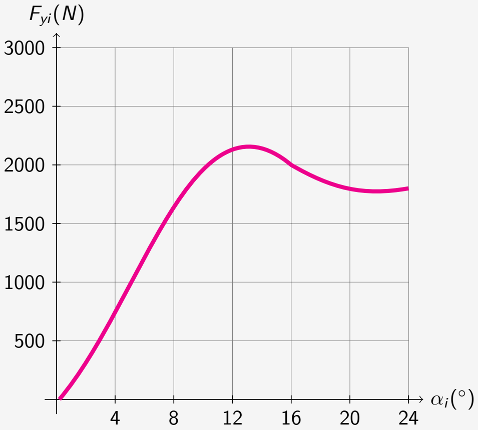
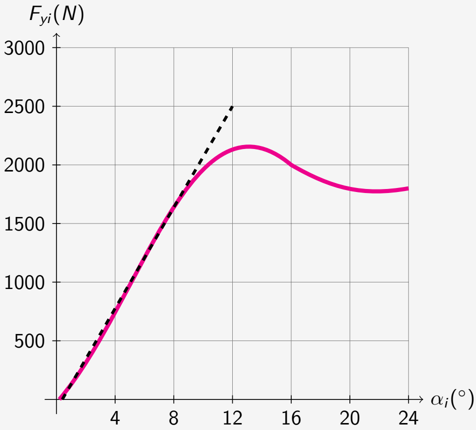


Problem Statement

- Pacejka's magic formula: Nonlinear model;
- Nominal conditions & small sideslip angles: c_i fixed

$$F_{yi} = c_i \alpha_i$$

c_i denotes the cornering stiffness of tires.



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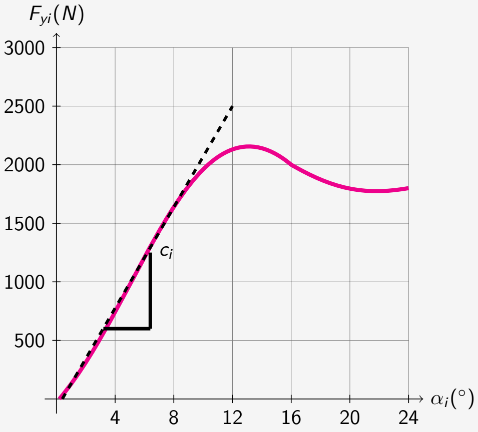

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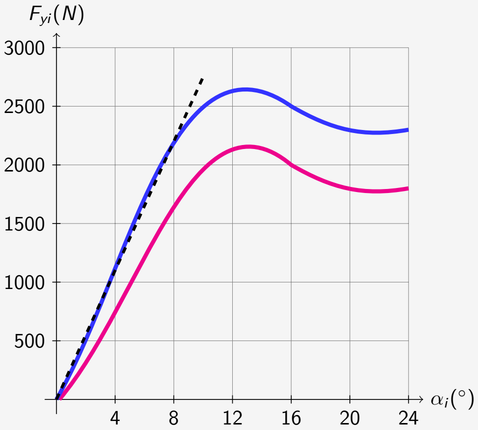

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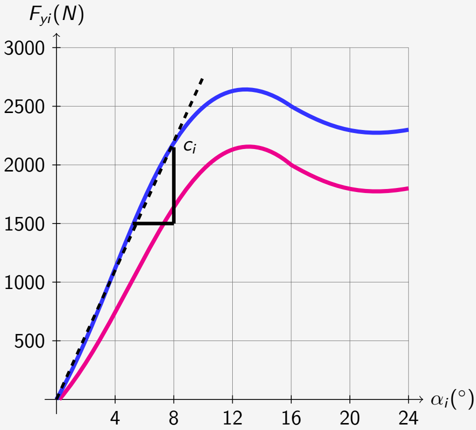

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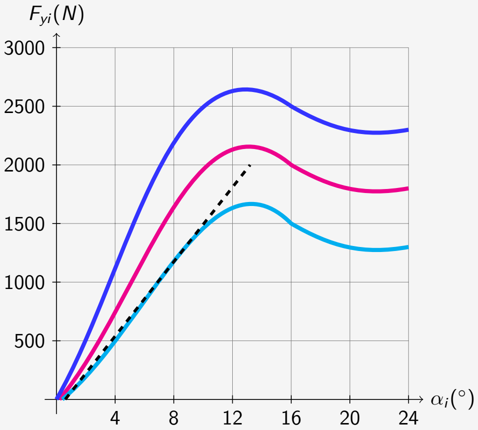

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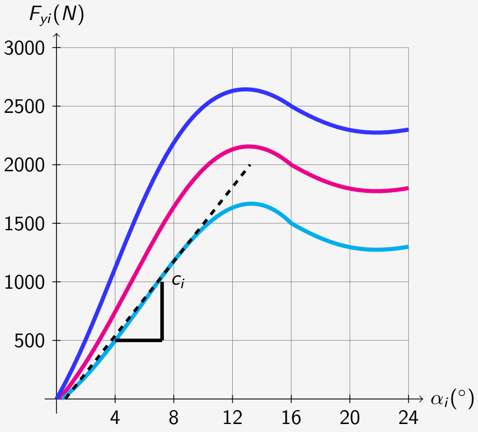
  c_i denotes the cornering stiffness of tires.
- Change on road conditions or nonlinear region is reached: c_i variable
- In practice, the cornering stiffness coefficients are not constant but time varying.



Proposed Approach

- Existing approaches $\rightarrow$ Cornering stiffness parameters are constants

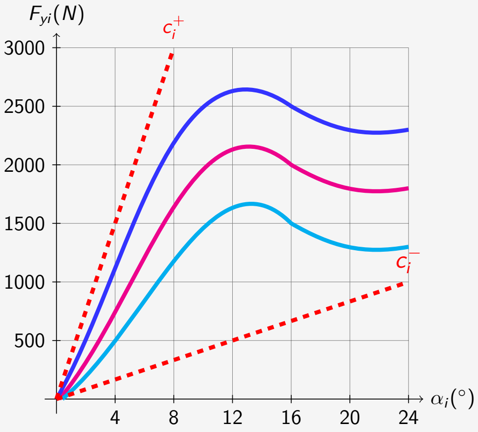
$$F_{yi} = c_i \alpha_i$$

- **Proposed approach** $\rightarrow$ Cornering stiffness parameters are uncertain

$$F_{yi} = (c_{i0} + \Delta c_i)\alpha_i$$

- **Assumption**:

$$\Delta c_i^- \leq \Delta c_i \leq \Delta c_i^+$$



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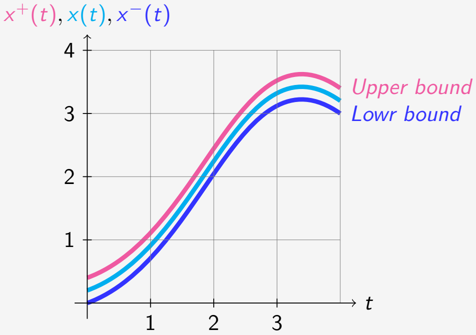
- Interval Observers: Under assumptions of knowing bounds on uncertain terms and initial conditions $\rightarrow$ Estimation of a feasible solution set of vehicle lateral velocity and yaw rate;



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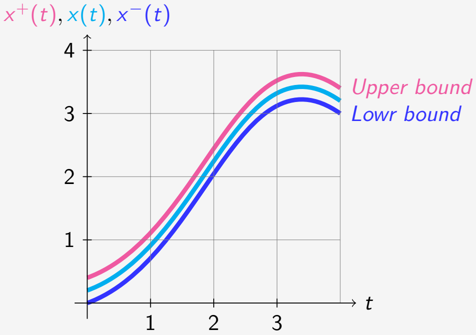

✎ **Main contribution.** A new estimation process for vehicle's lateral velocity and yaw rate presenting many benefits over the existing state of art works, within the dynamic estimation framework.

System Description

- Vehicle Lateral Dynamic model:

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\frac{c_f + c_r}{m v_x} & \frac{c_r l_r - c_f l_f}{m v_x} - v_x \\ \frac{c_r l_r - c_f l_f}{I_z v_x} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v_x} \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} + \begin{bmatrix} \frac{c_f}{m} \\ \frac{c_f l_f}{Iz} \end{bmatrix} \delta_f \quad (3)$$

where longitudinal velocity and cornering stiffness are treated respectively as the measurable and unmeasurable time varying parameters.

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- LPV state-space model

$$\begin{cases} \dot{x}(t) = A(\rho(t), \xi(t))x(t) + B(\xi(t))u(t) \\ y(t) = Cx(t) \end{cases} \tag{4}$$

where $\rho(t) = \begin{bmatrix} \frac{1}{v_x} & v_x \end{bmatrix}^T$ and $\xi(t) = \begin{bmatrix} c_r & c_f \end{bmatrix}^T$.

System Description

- Adopting a switching strategy based on longitudinal velocity variation range, a switched linear parameter-varying model for the vehicle lateral dynamics is derived

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(\rho(t), \xi(t))x(t) + B(\xi(t))u(t) \\ y(t) = Cx(t) \end{cases} \quad (5)$$

$\sigma(t) : \mathbb{R}_+ \rightarrow \mathcal{I} : \{1, \ldots, N\}$ is a Switching law that indicates at each time which mode is active.

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- Takagi-Sugeno (T-S) switched system

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))A^j_{\sigma(t)}(\xi(t))x(t) + B(\xi(t))u(t) \\ y(t) = Cx(t) \end{cases} \tag{6}$$

where $\rho(t)$ is the decision variable and $h^j_{\sigma(t)}(\rho(t))$ are switched weighting functions, $\forall j \in \{1, \ldots, 4\}$.

- The activating functions $h^j_{\sigma(t)}(\rho(t))$ satisfy the convex sum properties

$$\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t)) = 1, \quad 0 \leq h^j_{\sigma(t)}(\rho(t)) \leq 1 \tag{7}$$

Background on Interval observer design

Before state the main results...

[1] Blanchini, F., Colaneri, P., & Valcher, M. E. (2015). Switched positive linear systems. Foundations and Trends® in Systems and Control, 2(2), 101-273.

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- For any two vectors x_1, x_2 or matrices M_1, M_2 the inequalities $x_1 \leq x_2$, $x_1 \geq x_2$, $M_1 \leq M_2$ and $M_1 \geq M_2$ must be interpreted element-wise.
- A real matrix A_i, $\forall i \in \mathcal{I}$ is called a **Metzler matrix** if all its elements outside the main diagonal are positive, i.e,

$$\exists \beta \geq 0, \quad A_i + \beta \mathcal{I}_n \geq 0 \tag{8}$$

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An important application of positive dynamics

Lemma 1. Positive Switched Systems

For a Metzler matrix A_i, $\forall i \in \mathcal{I}$, the switched system

$$\dot{x}(t) = A_{\sigma(t)} x(t) + \delta_{\sigma(t)}(t) \tag{9}$$

is said to be a positive switched system [1] if $x(t_0) \geq 0$, A_i is a $n \times n$ Metzler matrix and $\delta_i(t) \geq 0$ $\forall i \in \{1, ..., N\}$.

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- **Assumption 1.** The pair (A_i^{j+}, C) is detectable $\forall i \in \mathcal{I}$, $j \in \{1, \ldots, 4\}$, $t \geq 0$.

- **Assumption 2.** There exist known functions $u^-(t)$, $u^+(t) \in \mathbb{R}^m$ such that

$$u^-(t) \leq u(t) \leq u^+(t), \quad \forall t \geq t_0 \tag{10}$$

- **Assumption 3.** There exist known constants matrices A_i^{j+}, A_i^{j-}, B^+, B^- $\forall i \in \mathcal{I}$, $\forall j \in \{1, \ldots, 4\}$, $\forall \rho(t) \in \nabla_i$ and $\forall \xi(t) \in \Xi = \left[[c_r^-, c_r^+] \quad [c_f^-, c_f^+] \right]^T$ such that:

$$\begin{aligned} A_i^{j-} &\leq A_i^j(\xi(t)) \leq A_i^{j+} \\ B^- &\leq B(\xi(t)) \leq B^+ \end{aligned}$$

  The matrices A_i^{j-}, A_i^{j+}, B^+ and B^- can be directly calculated using the known subset Ξ.

- **Notations**. $A_{\sigma(t)}(\rho(t), \xi_0) \longmapsto A_{\sigma(t),\rho,\xi_0}$.

Switched Interval Observer Design ☞

Theorem 1.

Assuming that the trajectory of system (6) is bounded $\|x\| \leq \mathcal{X}, \quad \forall t \geq t_0$. *Then, for all initial conditions* x_0 *such that* $x_0^- \leq x_0 \leq x_0^+$, *there exists a convergent switched interval observer of the TS model (6) of the form:*

$$\begin{cases} \dot{x}^+(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j+}_{\sigma(t)} x^+(t) + L^j_{\sigma(t)}(y - Cx^+(t)) + B^+ u^+(t) + \\ (A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0})(|x^+(t)| - x^+(t))) \\ \dot{x}^-(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j+}_{\sigma(t)} x^-(t) + L^j_{\sigma(t)}(y - Cx^-(t)) + B^- u^-(t) - \\ (A^{j+}_{\sigma(t)} - A_{\sigma(t),\rho,\xi_0})(|x^-(t)| + x^-(t))) \end{cases} \quad (11)$$

if the matrix $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^j_{\sigma(t)} C)$ *is* ***Metzler*** *and the matrix* $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j+}_{\sigma(t)} - L^j_{\sigma(t)} C)$ *is* ***Hurwitz*** $\forall \rho(t) \in \nabla_{\sigma(t)}$ *and* $\forall \xi(t) \in \Xi$.

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The upper estimation error $e^+(t) = x^+(t) - x(t)$ is governed by the following equation

$$\dot{e}^+(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))((A_{\sigma(t),\rho,\xi_0} - L^j_{\sigma(t)}C)e^+(t) + \delta^{j+}_{\sigma(t)}(t) \tag{12}$$

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where by construction $\delta^{j+}_{\sigma(t)}(t) \geq 0$.

Remark 1. It's clear that if $\sum_{j=1}^{4} h^{j}_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^{j}_{\sigma(t)}C)$ is Metzler then $\sum_{j=1}^{4} h^{j}_{\sigma(t)}(\rho(t))((A_{\sigma(t),\rho,\xi_0} - L^{j}_{\sigma(t)}C)$ is also Metzler for any $A_{\sigma(t),\rho,\xi_0}$ in the interval:

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Under Lemma 1, if $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^j_{\sigma(t)} C)$ is a Metzler Matrix, then the dynamics of $e^+(t)$ is positive, it follows that

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By the same reasoning, it follows that if $\sum_{j=1}^{4} h^{j}_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^{j}_{\sigma(t)}C)$ is Metzler, then the lower estimation error $e^{-}(t) = x(t) - x^{-}(t) \geq 0$ ➡ $x^{-}(t) \leq x(t)$, implies that

$$x^{-}(t) \leq x(t) \leq x^{+}(t)$$

By the same reasoning, it follows that if $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^j_{\sigma(t)}C)$ is Metzler, then the lower estimation error $e^-(t) = x(t) - x^-(t) \geq 0$ ➡ $x^-(t) \leq x(t)$, implies that

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➥ **Problem 1.**

Find the gain matrix $L^j_{\sigma(t)}$ such that $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j-}_{\sigma(t)} - L^j_{\sigma(t)}C)$ is Metzler $\forall j \in \{1, \ldots, 4\}, \forall \sigma(t)$.

- ② **Sufficient conditions for convergence**

The dynamics of the total error $e(t) = x^+(t) - x^-(t)$ is given by

$$\dot{e}(t) = \sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t)) \left((A^{j+}_{\sigma(t)} - L^j_{\sigma(t)} C) e(t) + \delta^j_{\sigma(t)}(t) \right) \tag{13}$$

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where by construction $\delta^j_{\sigma(t)}(t) \geq 0$.

➥ Problem 2.

Find the gain matrix $L^j_{\sigma(t)}$ such that $\sum_{j=1}^{4} h^j_{\sigma(t)}(\rho(t))(A^{j+}_{\sigma(t)} - L^j_{\sigma(t)} C)$ is Input-to-State Stable with respect to $\delta^j_{\sigma(t)}(t)$.

- The closed-loop stability is studied using a **Switched Fuzzy ISS-Lyapunov Function**

$$V(e(t)) = \sum_{i=1}^{N}\sum_{j=1}^{4} \lambda_i(t) h_i^j(\rho(t)) e^T(t) P_i^j e(t) \quad (14)$$

where $\lambda(t)$ represent the indicator function specifying the current active subsystem and P_i^j represent the i-th diagonal positive matrix.

- These properties are satisfied

$$\lambda_i(t) \geq 0, \ \ \forall i \in \mathcal{I}, \quad \sum_{i=1}^{N} \lambda_i(t) = 1, \quad \sum_{i=1}^{N} \dot{\lambda}_i(t) = 0$$

$$\sum_{i=1}^{N}\sum_{k=1}^{4} \lambda_i(t) \dot{h}_i^k(\rho(t)) = 0$$

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$$\sum_{i=1}^{N} \sum_{k=1}^{4} \lambda_i(t) \dot{h}_i^k(\rho(t)) = 0$$

- It can be shown that:

$$\dot{V}_i(e(t)) < -\varepsilon V_i(e(t)) + \gamma \sum_{i=1}^{N} \sum_{j=1}^{4} \lambda_i(t) h_i^j(\rho(t)) {\delta_i^j}^T(t) \delta_i^j(t) \tag{15}$$

- The closed-loop stability is studied using a **Switched Fuzzy ISS-Lyapunov Function**

$$V(e(t)) = \sum_{i=1}^{N} \sum_{j=1}^{4} \lambda_i(t) h_i^j(\rho(t)) e^T(t) P_i^j e(t) \qquad (14)$$

where $\lambda(t)$ represent the indicator function specifying the current active subsystem and P_i^j represent the i-th diagonal positive matrix.

- These properties are satisfied

$$\lambda_i(t) \geq 0, \ \ \forall i \in \mathcal{I}, \quad \sum_{i=1}^{N} \lambda_i(t) = 1, \quad \sum_{i=1}^{N} \dot{\lambda}_i(t) = 0$$

$$\sum_{i=1}^{N} \sum_{k=1}^{4} \lambda_i(t) \dot{h}_i^k(\rho(t)) = 0$$

- It can be shown that:

$$\dot{V}_i(e(t)) < -\varepsilon V_i(e(t)) + \gamma \sum_{i=1}^{N} \sum_{j=1}^{4} \lambda_i(t) h_i^j(\rho(t)) {\delta_i^j}^T(t) \delta_i^j(t) \qquad (15)$$

- Asymptotic stability is no longer ensured
- Convergence in a ball around the origin, to be minimized using ISS property

Theorem 2.

Assuming that

$$\sum_{i=1}^{N} \lambda_i(t) |\dot{h}_i^k(\rho(t))| \leq \sum_{i=1}^{N} \lambda_i(t) \phi_i^k \tag{16}$$

where $\phi_i^k \geq 0$ $(k = 1, .., 4)$ *are given scalars, if there exist, diagonal positive definite matrices* P_i^j, *matrices* W_i^j *and* M_i, $\forall i \in \mathcal{I}$, $j = \{1, \ldots, 4\}$, $\gamma > 0$ *for given positive scalars* ε *and* ϵ *such that the following conditions hold*

$$\min_{P_i^j, M_i, W_i^j} \gamma$$

$$P_i^j \succ 0 \tag{17}$$

$$P_i^k + M_i \succ 0 \tag{18}$$

...

$$\begin{bmatrix} \Lambda_i^j + \varepsilon P_i^j + \sum_{k=1}^{4} (\phi_i^k P_i^k + M_i) & P_i^j \\ P_i^j & -\gamma I_n \end{bmatrix} \prec 0 \tag{19}$$

$$P_i^j A_i^{j-} - W_i^j C + \epsilon P_i^j \geq 0 \tag{20}$$

where

$$\Lambda_i^j = {A_i^{j+}}^T P_i^j - C^T {W_i^j}^T + P_i^j A_i^{j+} - W_i^j C \tag{21}$$

Then the proposed observer can estimate the lower and upper bounds of the state vector $x(t)$ *for any switching signal, where* $L_i^j = {P_i^j}^{-1} W_i^j$.

Experimental validation

- The experimental data are acquired with a prototype vehicle;
- The run was performed on at test track located in the city of Versailles-Satory (France);
- The track is 3.5Km length with various curve profiles allowing vehicle dynamics excitation.

[2] This measure is not used for observer design. It serves only for observer estimation evaluation.

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Several sensors are implemented on the vehicle:

1. An inertial unit provide the yaw rate r measurement;
2. An absolute optical encoder to measure the steering angle δ_f;
3. An odometer to measure the vehicle longitudinal speed v_x;
4. A high precision Correvit sensor provide a measure of the sideslip angle [2].

[2] This measure is not used for observer design. It serves only for observer estimation evaluation.

Experimental validation

- The longitudinal velocity should be treated as a time-varying parameter;
- The cornering stiffness parameters are affected by 10% uncertainty in their nominal value.
- In this scenario, the lateral forces reach the nonlinear zone.

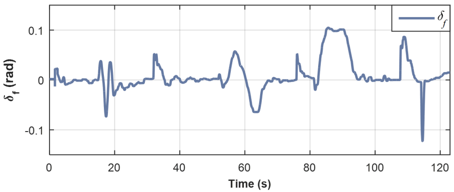

Figure: Steering angle.

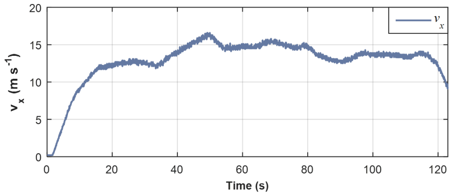

Figure: Longitudinal velocity.

Experimental validation

Consider the following switching law

$$\sigma(t) = \begin{cases} 1 & if \quad 0 < v_x \leq 6m.s^{-1} \\ 2 & if \quad 6m.s^{-1} < v_x \leq 11m.s^{-1} \\ 3 & if \quad 11m.s^{-1} < v_x \leq 16.6m.s^{-1} \end{cases} \tag{22}$$

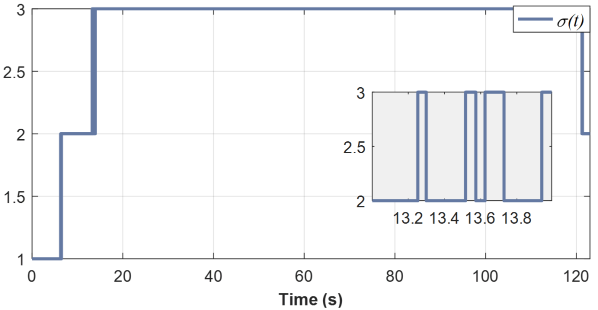

Figure: Switching signal $\sigma(t)$.

Experimental validation

Solving the linear matrix inequalities in theorem 2, gives the solutions

$$P_1^1 = \begin{bmatrix} 0.0080 & 0 \\ 0 & 0.0205 \end{bmatrix},\ P_1^2 = \begin{bmatrix} 0.0080 & 0 \\ 0 & 0.0205 \end{bmatrix},\ P_1^3 = \begin{bmatrix} 0.0080 & 0 \\ 0 & 0.3545 \end{bmatrix}$$

$$P_1^4 = \begin{bmatrix} 0.0080 & 0 \\ 0 & 0.3396 \end{bmatrix},\ P_2^1 = \begin{bmatrix} 0.2891 & 0 \\ 0 & 0.4252 \end{bmatrix},\ P_2^2 = \begin{bmatrix} 0.2891 & 0 \\ 0 & 0.4465 \end{bmatrix}$$

$$P_2^3 = \begin{bmatrix} 0.2870 & 0 \\ 0 & 0.5521 \end{bmatrix},\ P_2^4 = \begin{bmatrix} 0.2870 & 0 \\ 0 & 0.5521 \end{bmatrix},\ P_3^1 = \begin{bmatrix} 0.0673 & 0 \\ 0 & 0.3219 \end{bmatrix}$$

$$P_3^2 = \begin{bmatrix} 0.0673 & 0 \\ 0 & 0.3171 \end{bmatrix},\ P_3^3 = \begin{bmatrix} 0.0673 & 0 \\ 0 & 0.3407 \end{bmatrix}, P_3^4 = \begin{bmatrix} 0.0673 & 0 \\ 0 & 0.3407 \end{bmatrix}$$

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$$L_1^1 = 10^3 \begin{bmatrix} -0.0136 \\ 4.3830 \end{bmatrix},\ L_1^2 = 10^3 \begin{bmatrix} -0.0078 \\ 4.3813 \end{bmatrix},\ L_1^3 = \begin{bmatrix} -70.1966 \\ 260.5428 \end{bmatrix},\ L_1^4 = \begin{bmatrix} -68.2978 \\ 272.1352 \end{bmatrix}$$

$$L_2^1 = \begin{bmatrix} -11.1729 \\ 153.0217 \end{bmatrix},\ L_2^2 = \begin{bmatrix} -6.2345 \\ 145.5492 \end{bmatrix},\ L_2^3 = \begin{bmatrix} -12.6483 \\ 120.0691 \end{bmatrix},\ L_2^4 = \begin{bmatrix} -7.7325 \\ 119.1450 \end{bmatrix}$$

$$L_3^1 = \begin{bmatrix} -19.1574 \\ 331.7307 \end{bmatrix},\ L_3^2 = \begin{bmatrix} -13.6983 \\ 336.8248 \end{bmatrix},\ L_3^3 = \begin{bmatrix} -19.5678 \\ 314.5372 \end{bmatrix},\ L_3^4 = \begin{bmatrix} -14.0961 \\ 316.8263 \end{bmatrix}$$

Experimental validation

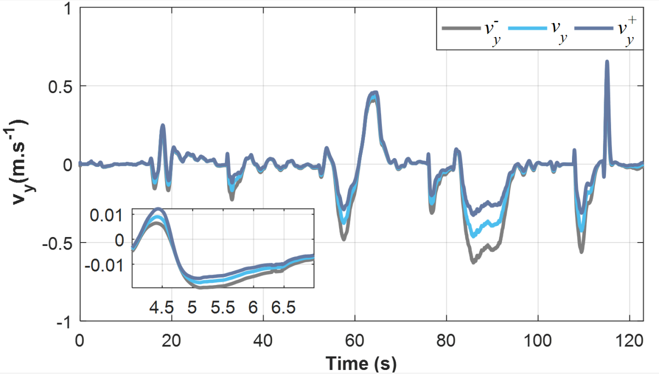

Figure: Interval observer of Lateral velocity.

Experimental validation

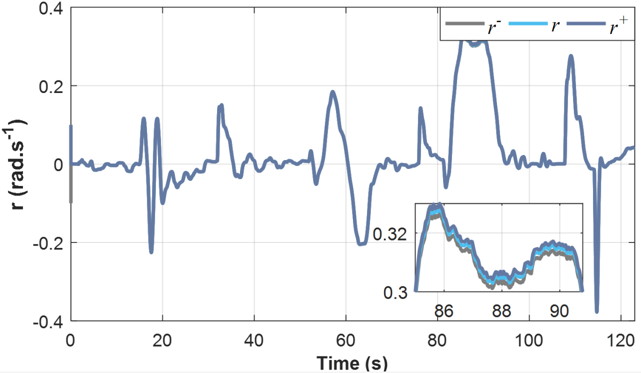

Figure: Interval observer of yaw rate.

Conclusion

- Robust estimation of lateral velocity and yaw rate using interval observers;
- Vehicle model subject to interval uncertainties (cornering stiffness & longitudinal velocity);
- The simulation results demonstrate the validity of the proposed approach.
- The convergence time is short and the intervals width are tight.

Thank you for your attention!

Robust estimation of vehicle lateral velocity and yaw rate using Switched T-S Fuzzy Interval Observer

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